EXERCISING IN K-THEORY: BRANE CONDENSATION WITHOUT TACHYON

CORNELIU SOCHICHIU

ABSTRACT. We show that the p-dimensional noncommutative Yang–Mills model corresponding to a $(p-1)$-brane allows solutions which correspond to lower branes. This may be interpreted as the Morita equivalence of noncommutative planes of various dimensions.

1. INTRODUCTION

Quantum field theory on noncommutative spaces [1], proved to be an useful and powerful tool in the study of nonperturbative strings [2].

Thus, the dynamics of branes in the presence of nonzero antisymmetric field $B_{\mu\nu}$ is described by noncommutative gauge models. In the limit of large $B_{\mu\nu}$ the noncommutativity parameter $\theta^{\mu\nu}$, is given by the inverse matrix, $\theta^{\mu\nu} = B_{\mu\nu}^{-1}$.

In this approach the brane and string degrees of freedom are expressed in terms of the respective noncommutative model. A definite progress was made in understanding these models, in special, their classical solutions related to the brane condensation (see e.g. [3, 4, 5, 6]).

These models exhibit new and unexpected properties in comparison with their commutative counterparts. This some people refer to as *the magic of noncommutativity*. The noncommutative solitonic solutions, which possess no analogues in the commutative world have the interpretation in terms of condensation of unstable Dp-branes to lower dimensional ones by collapsing of certain extensions of the unstable brane.

Another manifestation of this magic is the Morita equivalence which is an equivalence relation between different noncommutative spaces. It is believed that Morita equivalent spaces correspond to physically equivalent situations [7, 8, 9, 10, 11, 12].

In earlier papers [13, 14] it was proposed that a p-dimensional noncommutative Yang–Mills model with scalars can manifest itself as a Yang–Mills model with scalar fields in a different dimension. This phenomenon allows one to claim some equivalence relations between some noncommutative gauge models in various dimensions. In the actual work we further elaborate on this equivalence relation and claim that this relation can be interpreted in some sense as a Morita equivalence. Also we propose a demonstration of this equivalence, which in our opinion is a more natural than one proposed by the author of [14], since it does not require additional alteration of the noncommutative plane.

The solutions realising this equivalence can also be interpreted in terms of brane condensation. As in the case of tachyon mediated brane condensation the number of "degrees of freedom" of collapsed and non-collapsed brane is the same, however, in contrast to this in our case the respective dimensions collapse to zero size.

The plan of the paper is as follows. In the next section we give an alternative construction for the solution relating p-dimensional noncommutative Yang–Mills model with d scalar fields to $p = 2$-dimensional Yang–Mills model with $d + p - 2$ scalar fields. (The total number of fields $D = p + d$ is kept fixed.) In the third

section we build the same correspondence in terms of K-theory. Finally we discuss the results.

2. The Equivalence

In this section we find a solution in the p-dimensional noncommutative Yang–Mills model with scalar fields, which corresponds to a two-dimensional Yang–Mills model with scalar fields. We conventionally call this model the "Yang–Mills–Higgs model" and hope that there will be no confusion regarding this notion.

Consider the model of p-dimensional noncommutative U(1) Yang–Mills field interacting with d real (Hermitian) scalar fields ϕ_i $i = 1, \dots, d$, and living on non-commutative space given by the algebra

$$(1) \qquad\qquad [x^\mu, x^\nu] = \mathrm{i}\theta^{\mu\nu},$$

where we assume that the antisymmetric matrix $\theta^{\mu\nu}$ is invertible.

The model is described by the action,

$$(2) \qquad\qquad S = \int \mathrm{d}^p x \left(-\frac{1}{4g^2} F_{\mu\nu}^2 - \frac{1}{2}(\nabla_\mu \phi_i)^2 + \frac{1}{4g^2}[\phi_i, \phi_j]^2 \right),$$

where,

$$(3) \qquad\qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \mathrm{i}(A_\mu * A_\nu - A_\nu * A_\mu).$$

The star product in eqs. (3) is defined as follows,

$$(4) \qquad\qquad (A * B)(x) = \left. \mathrm{e}^{-\frac{\mathrm{i}}{2}\theta^{\mu\nu}\partial_\mu \partial'_\nu} A(x)B(x') \right|_{x'=x},$$

∂_μ and ∂'_μ denote derivatives with respect to x^μ and x'^μ.

The functions on noncommutative space, subject to star product (4) realise the representation of the Heisenberg algebra (1) in terms of Weyl ordered symbols.

One can, however, come back to the operator form. In this case the action (2) is rewritten in the form as follows,

$$(5) \qquad S = (2\pi)^{\frac{p}{2}} |\operatorname{Pf}\theta| \frac{1}{4g^2} \operatorname{tr}([X_M, X_N]^2 - B^2) = \frac{1}{4\tilde{g}^2} \operatorname{tr}([X_M, X_N]^2 - B^2),$$

where capital roman indices M, N span the range $1, \dots, D = p + d$. $\operatorname{Pf}\theta$ stands for the Pfaffian of the matrix $\theta^{\mu\nu}$, $B^2 = (\theta_{\mu\nu}^{-1})^2$, $X_\mu = \theta_{\mu\nu}^{-1} x^\nu + A_\mu$ and $X_i = \phi_i$ are Hermitian operators acting on the Hilbert space $\mathcal{H}$ on which the Heisenberg algebra (1) is represented. With the background invariant coupling $\tilde{g} = g/\sqrt{(2\pi)^{p/2}|\operatorname{Pf}\theta|}$ this corresponds to the bosonic part of the IKKT matrix model [15], at $N = \infty$.

Another advantage of the form (5) of the action is that it is written in the background independent form [16], and Yang–Mills–Higgs system with the same number of fields look similar in different dimensions. In what follows we are going to show that noncommutative Yang–Mills–Higgs models in different dimensions are just perturbative sectors related to different backgrounds of the same model given by the action (5).

As usual, by a proper Lorentz transformation one can bring the "tensor" $\theta^{\mu\nu}$ to the canonical block-diagonal form with i-th, $i = 1, \dots, p/2$, 2×2 antisymmetric block having $\theta_{(i)}$ as its entry. In this case the set of operators x^μ is split in momentum and position operators p_i and q^i satisfying the usual Heisenberg commutation relations

$$(6) \qquad\qquad [p_i, q^j] = -\mathrm{i}\theta_{(i)}\delta_i^j, \qquad \theta_{(i)} > 0.$$

Further, one can pass to "complex coordinates" a_i, $\bar{a}_i$, which are given by oscillator lowering and rising operators,

$$(7) \qquad a_i = \frac{1}{\sqrt{2\theta_{(i)}}}(q^i + \mathrm{i}p_i), \qquad \bar{a}_i = \frac{1}{\sqrt{2\theta_{(i)}}}(q^i - \mathrm{i}p_i),$$

$$(8) \qquad [a_i, \bar{a}_j] = \delta_{ij}, \qquad N_i = \bar{a}_i a_i,$$

where in the last equation no sum is assumed. Eigenvalues of N_i form an $p/2$-dimensional half-infinite lattice,

$$(9) \qquad N_i \left|\vec{n}\right\rangle = n_i \left|\vec{n}\right\rangle, \qquad \vec{n} \in \mathbb{Z}_+^{\frac{p}{2}},$$

where $\bar{a}_i$ and a_i act as rising and lowering operators for the value n_i.

Equations of motion corresponding to the action (5) look as follows,

$$(10) \qquad [X_M, [X^M, X^N]] = 0.$$

In this viewpoint X_M are just a set of p operators acting on the Hilbert space $\mathcal{H}$ with basis formed by vectors $\left|\vec{n}\right\rangle$. In fact an arbitrary solution to the eq. (10) can be interpreted either as (flat) covariant derivatives of some noncommutative space or a constant curvature field configuration in p dimensions.

As an example consider the case $p = 4$ and $d = 0$. The "complex coordinates" are a_i and $\bar{a}_i$, $i, j = 1, 2$. In fact, one can consider $d \neq 0$, but the four-dimensional scalar fields do not play any important rôle in the analysis at this stage.

The equation (10) has a solution,

$$(11) \qquad X_1 = \sqrt{\frac{\theta}{2}}(A + \mathrm{i}\bar{A}), \qquad X_2 = \sqrt{\frac{\theta}{2}}(A - \mathrm{i}\bar{A})$$

$$(12) \qquad X_3 = X_4 = \text{constant},$$

where the operators A and $\bar{A}$ and the parameter θ are defined as follows.

Consider e.g. an oriented zigzag line starting from the origin and filling the two-dimensional quarter-infinite lattice, like one depicted in Fig.1. Relabel the lattice point and the respective eigenvector by the integer value of the length of the zigzag line from the origin to this point. That is,

$$(13) \qquad n \mapsto \vec{n}$$

$$(14) \qquad \left|\vec{n}\right\rangle \mapsto \left|n\right\rangle = \left|\vec{n}(n)\right\rangle.$$

Since the zigzag passes each point no more (and no less) than once therefore this relabels the eigenvectors in a unique way. Operators A and $\bar{A}$ are defined by,

$$(15) \qquad A\left|n\right\rangle = \sqrt{n}\left|n-1\right\rangle,$$

$$(16) \qquad \bar{A}\left|n\right\rangle = \sqrt{n+1}\left|n+1\right\rangle,$$

$$(17) \qquad [A, \bar{A}] = 1,$$

where corresponding to the Figure 1, A is moving towards the origin (against the arrows on the picture) and $\bar{A}$ moving from origin (along the arrows). Operators A and $\bar{A}$ can be expressed as functions on (normal symbols of) a_i and $\bar{a}_i$. We do not know the exact analytic expressions for this functions corresponding to the case depicted in Fig.1, fortunately we do not need it.

Requiring finiteness of the action (5) computed on the solution (15), (16) one has for the noncommutativity parameter, $\theta = \sqrt{2/(B^2)}$.

Having solution (13-16) one can introduce the Weyl ordered symbol with respect to A and $\bar{A}$ for some well behaved operator Φ, e.g. one satisfying $\mathrm{tr}\,\Phi^*\Phi < \infty$.
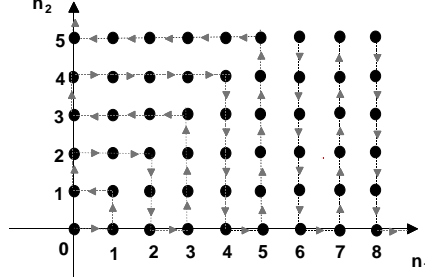
FIGURE 1. An example of the two-dimensional solution in the four-dimensional theory. The orbit of the operator $\bar{A}$ is shown by arrows, while A act counter the arrows. Operators a_i and $\bar{a}_i$, $i = 1, 2$ act along respective axes. This is only one of an infinite number of possibilities to enumerate sites of the lattice (n_1, n_2).

The respective Weyl symbol $\Phi(A, \bar{A})$ defines a function on the *two-dimensional* noncommutative plane,

$$(18) \qquad \Phi(A, \bar{A}) = \frac{1}{2\pi} \int \mathrm{d}^2 k \ \mathrm{e}^{2\mathrm{i}(\bar{A}k + \bar{k}A)} \operatorname{tr}(\mathrm{e}^{-2\mathrm{i}(\bar{A}k + \bar{k}A)}\Phi),$$

$$(19) \qquad (\Phi * \Psi)(A, \bar{A}) = \mathrm{e}^{-\frac{1}{2}(\partial\bar{\partial}' - \bar{\partial}\partial')}\Phi(A, \bar{A})\Psi(A', \bar{A}') \Big|_{\substack{A'=A \\ \bar{A}'=\bar{A}}},$$

where the integration in (18) is performed through the two-dimensional (commutative) complex plane $(k, \bar{k})$.

This gives formulas for passing from functions on the four-dimensional noncommutative plane to functions on the two-dimensional one. By construction this procedure is invertible and therefore it establishes an equivalence relation two algebras of noncommutative functions [14]. In next section we are going to show that this is in fact a Morita type equivalence.

The solution (13-16) is at no way unique. All possible solutions of this type are parameterised by different ways in choosing ordered basis in the Hilbert space.

In the case of arbitrary even $p \leq D$ one obtains such a solution by enumerating the basis of the separable Hilbert space and defining A and $\bar{A}$ according to (15) and (16) where n is the number of the basis element.

The arbitrariness in choosing an orthonormal basis in the Hilbert space $\mathcal{H}$ is parameterised by the unitary operator $U \in \mathcal{U}(\mathcal{H})$, where $\mathcal{U}(\mathcal{H})$ is the unitary group of the Hilbert space $\mathcal{H}$, sometimes called $\mathrm{U}(\infty)$. Therefore $\mathcal{U}(\mathcal{H})$ also is the moduli space of the map (13-16). As it is clear this dependence however can be eaten by a gauge transformation of either two-dimensional or p-dimensional model.

So far we the transformation of scalar noncommutative functions. For a non-scalar function e.g. a vector one but other than the gauge field one has, roughly speaking, two two-dimensional vector components and $(p - 2)$ scalars. The problem is how to split it in the vector and scalar components. A priori, there is no restriction to do this, the total arbitrariness being by the the Grassman manifold SO(p)/SO(2)×SO(p-2).

Having this one may be tempted to translate from p to two dimensions also the small fluctuations of the gauge fields A_μ. In trying to do this there is a problem. By

the above construction one can translate only quantities which transform covariantly under the action of the noncommutative U(1). (i.e. such quantities which can be represented by a background independent operator.) It is known that A_μ does not transform covariantly. However, the quantity $X_\mu = p_\mu + A_\mu$, $\mu = 1, \ldots, p$ do and can be written in the two dimensional form. But when one will try to go back from the description in terms of X's to the description in terms of two dimensional gauge fields and scalars one will realise that neither of the fields do vanish at the infinite, although the action is finite. This in fact means that there are different perturbative regimes corresponding to solutions giving different dimensionalities.

Let us note, that in order to keep the action (2) invariant under this redefinition one has to require,

$$(20) \qquad (2\pi)^{\frac{p}{2}} \operatorname{Pf} \theta_{(p)} \frac{1}{4g_{(p)}^2} = 2\pi\theta_{(2)} \frac{1}{4g_{(2)}^2},$$

where the subscript in the parentheses denotes the dimension to which the quantity refers. This condition gives for the two dimensional gauge coupling,

$$(21) \qquad g_{(2)}^2 = \frac{\sqrt{2}}{(2\pi)^{\frac{p}{2}-1} B \operatorname{Pf} \theta_{(p)}} g_{(p)}^2.$$

The arguments above can be turned back, i.e. one can consider the initial p dimensional noncommutative space (1) as a solution in the two dimensional model defined by (11) and (15,16). This was originally the way proceeded in ref [14].

We came thus to the natural conclusion that noncommutative Yang–Mills–Higgs models in different dimensions behave like different perturbative regions of the same background independent model (5) [16], defined in terms of operators acting on some abstract separable Hilbert space $\mathcal{H}$.

3. K-theory meaning

K-theory toolkit seems to be appropriate for the study of brane dynamics as well as of noncommutative gauge models [8, 9, 10, 11, 12]. In this section we are going to exploit some K-theory tools in order to understand the results of the previous section from this point of view. In order to use them let us make following [8, 9, 10, 11, 12] a very short review of K-theory in application to the noncommutative geometry.

Consider an associative complex algebra $\mathfrak{A}$ with involution "*" (a C^*-algebra). We will mainly think about the algebra of complex functions on the noncommutative plane. In our case it is the Heisenberg algebra. Let E be its left module i.e.,

$$(22) \qquad a(m) = am \in E, \qquad (a'a)(m) = a'(am) = a'am,$$

for arbitrary $m \in E$, and $a, a' \in \mathfrak{A}$. Right module is defined in a similar way but with consequent action of elements of $\mathfrak{A}$ from the right.

The algebra $\mathfrak{A}$ itself as well as n copies of it $\mathfrak{A} \oplus \mathfrak{A} \oplus \cdots \oplus \mathfrak{A}$ is a primitive example of both left and right modules, such modules are called *free*. A module E for which exists another module E' such that $E \oplus E'$ is free is called a *projective* one. (It is clear that E' is also a projective module.) The set of left or right projective modules form a semigroup with respect to the direct sum operation. This semigroup can be "upgraded" to a group as follows.

Consider pairs of modules (E, F), with the composition rule $(E, F) + (E', F') = (E \oplus E', F' \oplus F)$ and the equivalence relation $(E, F) \sim (E \oplus G, F \oplus G)$, for arbitrary module G. This equivalence classes already form a group whose unity is given by

(G, G)-pairs and the opposite element to (E, F) given by (F, E),

$$(E, F) + (F, E) = (E \oplus F, E \oplus F) \sim (G, G).$$

This trick is similar to one used to extend the set of positive numbers to real ones. The group one gets in a such way is called the $K(\mathfrak{A})$, or, if $\mathfrak{A}$ is the algebra of functions on some space $\mathcal{M}$ it is denoted as $K(\mathcal{M})$.

Let us equip our left or right projective module E, with an $\mathfrak{A}$-valued product $\langle\ ,\rangle_{\mathfrak{A}}$, satisfying,

$$(23) \qquad\qquad\qquad \langle m, m'\rangle_{\mathfrak{A}}^* = \langle m', m\rangle_{\mathfrak{A}}$$

$$(24) \qquad\qquad\qquad \langle am, m'\rangle_{\mathfrak{A}} = a\langle m, m'\rangle_{\mathfrak{A}}$$

$$(25) \qquad\qquad\qquad \langle m, m'\rangle_{\mathfrak{A}} \text{ is a positive element in } \mathfrak{A}.$$

The $\mathfrak{A}$-module E is called *full* when the linear span of the range of $\langle\ ,\rangle_{\mathfrak{A}}$ is dense in $\mathfrak{A}$.

One can introduce connection ∇_α on the E with respect to some element of the algebra of infinitesimal automorphisms of $\mathfrak{A}$: $a \to a + \delta_\alpha a$, labelled by some element α, which satisfies,

$$(26) \qquad\qquad\qquad \nabla_\alpha(am) = a\nabla_\alpha(m) + (\delta_\alpha a)m,$$

and it is linear in α. Using this connection one can built the curvature associated to it,

$$(27) \qquad\qquad\qquad F_{\alpha\beta} = [\nabla_\alpha, \nabla_\beta] - \nabla_{[\alpha,\beta]}.$$

$\mathfrak{A}$-linear maps $T : E \to E$ which have an adjoint with respect to the product (23-25) and commute with the action of $\mathfrak{A}$ on E form the algebra $\operatorname{End}_{\mathfrak{A}} E$ of endomorphisms of the $\mathfrak{A}$-module E.

By definition an algebra $\mathfrak{B}$ is Morita equivalent to $\mathfrak{A}$ if it is isomorphic to $\operatorname{End}_{\mathfrak{A}} E$ for some complete module E.

There exists the following criterium for Morita equivalence of two algebras $\mathfrak{A}$ and $\mathfrak{B}$. A left $\mathfrak{A}$-module P which is also a right $\mathfrak{B}$-module is called $(\mathfrak{A}, \mathfrak{B})$-bimodule. Assume that P as $\mathfrak{A}$- and $\mathfrak{B}$-module is equipped with $\mathfrak{A}$-valued product $\langle\ ,\rangle_{\mathfrak{A}}$, and $\mathfrak{B}$-valued product $\langle\ ,\rangle_{\mathfrak{B}}$, and it is full as both $\mathfrak{A}$- and $\mathfrak{B}$-module. When it exists such a module is called $(\mathfrak{A}, \mathfrak{B})$ *equivalence bimodule*, in this case algebras $\mathfrak{A}$ and $\mathfrak{B}$ are Morita equivalent. The Morita equivalence allows one to establish relations between various structures of the equivalent algebras and their modules, like endomorphisms, connections, etc.

It is conjectured [8, 9, 10, 11, 12], that Morita equivalent algebras in string theory correspond to physically equivalent systems e.g. related by duality transformations. In noncommutative theory the gauge models on the dual tori are also known to be Morita equivalent [7, 9].

Let us return back to the model (5). The algebra $\mathfrak{A}_p$ now is one generated x^μ subject to commutation relation (1), or in alternative basis, respectively, by $p/2$-dimensional oscillator rising and lowering operators $\bar{a}_i, a_i$ (7,8). We will use the last choice. In this case the $p/2$-dimensional oscillator Hilbert space $\mathcal{H}_p$ with the basis $|\vec{n}\rangle$ plays the role of a complete $\mathfrak{A}$-module.

Consider the space $P = \mathcal{H}_2 \otimes \mathcal{H}_p^*$ which is the linear span of elements $|n\rangle\langle\vec{n}|$, where $|\vec{n}\rangle \in \mathcal{H}_2$ and $\langle\vec{n}| \in \mathcal{H}_p^*$. P is at the same time left module for one-dimensional oscillator algebra (two-dimensional noncommutative functions) and right module for the $p/2$ dimensional oscillator algebra (p-dimensional noncommutative plane function algebra). As both $\mathfrak{A}_p$ and $\mathfrak{A}_2$ module P is complete, hence it is an equivalence one. Therefore, the function algebra of the p-dimensional noncommutative

plane is in some sense[1] Morita equivalent to one on the two-dimensional noncommutative plane.

In the case of Morita equivalent algebras one has a correspondence between the $\mathfrak{A}_p$ and $\mathfrak{A}_2$ modules. Thus for an $\mathfrak{A}_p$-module $\mathcal{H}_p$ one has an $\mathfrak{A}_2$-module $\mathcal{H}_2$ given by,

$$(28) \qquad \mathcal{H}_2 = P \otimes_{\mathfrak{A}_p} \mathcal{H}_p,$$

where the tensor product with respect to $\mathfrak{A}_p$ is obtained from usual (complex) tensor product $\otimes$ by means of identification, $pa \otimes m \sim p \otimes am$, $a \in \mathfrak{A}_p$.

It seems to be a problem because neither of modules $\mathcal{H}_p$ or P seems to be *finitely generated projective*. However, one can avoid this problem by considering a *regularised* system. One can regularise the algebras (7,8) and (15,16), e.g. by q-deforming them as proposed in [17] with $q^N = 1$,

$$(29) \qquad a_i \left| \vec{n} \right\rangle = \sqrt{\frac{1}{\pi} \sin \frac{\pi n_i}{N}} \left| \vec{n} - \vec{e}_i \right\rangle, \qquad \bar{a}_i \left| \vec{n} \right\rangle = \sqrt{\frac{1}{\pi} \sin \frac{\pi (n_i + 1)}{N}} \left| \vec{n} + \vec{e}_i \right\rangle,$$

where $\vec{e}_i$ is the i-th unit lattice vector. Then the limit $N \to \infty$ corresponds to the "cut-off" removing. In this case the regularised Hilbert space and the equivalence module become finite dimensional and finitely generated projective. Moreover, irrelevant to N all regularised models fall in the same Morita equivalence class and one can define the cut-off removed model as an extremal element of the extremal element of this class.

However, as noted in [12], there is a difference between finite dimensional cases and the case $N = \infty$. In particular all infinite dimensional separable Hilbert spaces are known to be isomorphic. In this case establishing Morita equivalence is equivalent to establishing all possible maps between the $\mathfrak{A}$ and $\mathfrak{B}$ modules.

Indeed, due to the irreducibility of the action of the Heisenberg algebra $\mathfrak{A}_p$ on the Hilbert space, the equivalence module P can be represented as a tensor product $\mathcal{H}_2 \otimes \mathcal{H}_p^*$ which is isomorphic to $\mathrm{Hom}(\mathcal{H}_p, \mathcal{H}_2)$. This set contains all maps from the Hilbert space of the p-dimensional Heisenberg algebra to the Hilbert space of the 2-dimensional one. In the previous section we considered only isomorphic maps: $\mathrm{Iso}(\mathcal{H}_p, \mathcal{H}_2) \subset \mathrm{Hom}(\mathcal{H}_p, \mathcal{H}_2)$. Since it is Hilbert spaces which are mapped, the set of isomorphic maps preserving the Hilbert space product is in its turn isomorphic to the infinite dimensional unitary group: $\mathrm{Iso}(\mathcal{H}_p, \mathcal{H}_2) \cong \mathcal{U}(\mathcal{H}_p) \cong \mathcal{U}(\mathcal{H}_2)$.

This gives exactly the moduli of the equivalence of p- and 2-dimensional models described in the previous section.

4. Discussions and Conclusions

In this paper we have shown that a p-dimensional noncommutative gauge model with scalar fields possesses solutions which can be interpreted as a 2-dimensional gauge model. We considered nondegenerate solutions, i.e. ones which realise a isomorphism (equivalence) between two models. The moduli of such solutions are given by the group of unitary transformations of the separable Hilbert space. This means that the isomorphism is unique up to a noncommutative U(1) gauge transformation.

The construction of the second section can be turned back, i.e. one can consider solutions in the 2-dimensional gauge model with scalars which behaves like a $p > 2$-dimensional model with less scalars. This suggests that the equivalence is valid for all noncommutative models of the types given by the action (2) in even dimensions less or equal D, which is the total number of fields and with the same factor $B^2 = (\theta_{\mu\nu}^{-1})^2$.

---

[1]It seems that the Hilbert space itself is not finitely generated projective module.

The K-theory considerations allow one to generalise the above solutions and to consider arbitrary maps from the p-dimensional model to another p'-dimensional one and back while both p and p' are even. These include also the projector solutions, i.e. ones when the orbit of A and $\bar{A}$ span only a sublattice of the $p/2$-dimensional lattice of the eigenvalues of N_i. Such solutions should have some relevance to the noncommutative solitons [3, 4, 6, 18].

In the string theory picture the gauge fields present in the model we considered describe the tangential coordinates of a $(p-1)$-brane while the scalar fields correspond to the transversal ones. The interpretation of the obtained solutions, therefore is as a brane whose number of extensions collapsed to zero size or oppositely as some new dimensions have been blown up. This is similar to the tachyon condensation picture, however, there is no tachyonic mode in the spectrum of the model since we are considering BPS and therefore stable solutions. The other difference is that in our case the the thickness of a condensed brane is exactly zero and not finite one as in the tachyon condensation case. Also, the tachyon condensation mechanism cannot provide appearance of new extensions to a brane in contrast to our case.

Due to the described equivalence we have a single operator model described by the action (5) with an abstract separable Hilbert space rather than noncommutative gauge models in different dimensions. The respective gauge models are given by fluctuations around particular solutions in the main operator model.

One may be surprised by the fact that the gauge models have different renormalisation behaviour in different dimensions. In particular one has different external divergence indices for the same Feynmann diagrams in different dimensions.

In fact there is no contradiction if to observe that the respective noncommutative models correspond to different points of perturbative expansion of the main operator model. It is usual that the expansion around different points may have different convergence properties, but due to the IR/UV mixing one may presume that nonperturbatively, or even up to all orders in perturbation theory the model has the same renormalisation behaviour in all dimensions.

References

[1] A. Connes, *A short survey of noncommutative geometry*, J. Math. Phys. **41** (2000) 3832–3866, [`hep-th/0003006`].

[2] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, JHEP **09** (1999) 032, [`hep-th/9908142`].

[3] R. Gopakumar, S. Minwalla, and A. Strominger, *Noncommutative solitons*, JHEP **05** (2000) 020, [`hep-th/0003160`].

[4] C. Sochichiu, *Noncommutative tachyonic solitons: Interaction with gauge field*, JHEP **08** (2000) 026, [`hep-th/0007217`].

[5] R. Gopakumar, S. Minwalla, and A. Strominger, *Symmetry restoration and tachyon condensation in open string theory*, `hep-th/0007226`.

[6] M. Aganagic, R. Gopakumar, S. Minwalla, and A. Strominger, *Unstable solitons in noncommutative gauge theory*, `hep-th/0009142`.

[7] A. Connes, M. R. Douglas, and A. Schwarz, *Noncommutative geometry and matrix theory: Compactification on tori*, JHEP **02** (1998) 003, [`hep-th/9711162`].

[8] P. Horava, *Type IIA D-branes, K-theory, and matrix theory*, Adv. Theor. Math. Phys. **2** (1999) 1373, [`hep-th/9812135`].

[9] A. Schwarz, *Morita equivalence and duality*, Nucl. Phys. **B534** (1998) 720–738, [`hep-th/9805034`].

[10] D.-E. Diaconescu, G. Moore, and E. Witten, *A derivation of K-theory from M-theory*, `hep-th/0005091`.

[11] D.-E. Diaconescu, G. Moore, and E. Witten, *E(8) gauge theory, and a derivation of K-theory from M-theory*, `hep-th/0005090`.

[12] E. Witten, *Overview of K-theory applied to strings*, `hep-th/0007175`.

[13] C. Sochichiu, *M(any) vacua of IIB*, JHEP **05** (2000) 026, [`hep-th/0004062`].

[14] C. Sochichiu, *On the equivalence of noncommutative models in various dimensions*, *JHEP* **08** (2000) 048, [`hep-th/0007127`].

[15] N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, *A large-N reduced model as superstring*, *Nucl. Phys.* **B498** (1997) 467, [`hep-th/9612115`].

[16] N. Seiberg, *A note on background independence in noncommutative gauge theories, matrix model and tachyon condensation*, *JHEP* **09** (2000) 003, [`hep-th/0008013`].

[17] E. E. Donets, A. P. Isaev, C. Sochichiu, and M. Tsulaia, *Brane vacuum as chain of rotators*, *JHEP* **12** (2000) 022, [`hep-th/0011090`].

[18] J. A. Harvey, P. Kraus, and F. Larsen, *Exact noncommutative solitons*, *JHEP* **12** (2000) 024, [`hep-th/0010060`].

Institutul de Fizică Aplicată AŞ, str. Academiei, nr. 5, Chişinău, MD2028 MOLDOVA

Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow Reg., RUSSIA
  E-mail address: `sochichi@thsun1.jinr.ru`